PULLING OF GLASS MICROCAPILLARIES: THEORY AND EXPERIMENT

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An approximate analytical solution is obtained which predicts a microcapillary form during pulling which is close to that observed in experiments.

Microcapillaries (hollow fibers) find wide application in opto- and radioelectronics, artificial kidney apparatuses, etc. In [1], an analytical theoretical model is proposed of pulling of glass microcapillaries from a tubular workpiece which is heated as it passes through a furnace. (Numerical solutions are also known of the problems on pulling of hollow fibers [2-4]; in the last work a dynamic problem is solved without consideration of heat transfer and viscosity variation with temperature.) In the present work the predictions made by the model [1] are compared with experimental data. As a result, it is possible to determine the configuration of a fiber narrowing zone in the pulling process and to evaluate its dimensions.

First of all we briefly dwell on the pulling model and its theoretical description. Figure 1 shows how a glass tube (a workpiece) is brought into the furnace, heated (as a result, glass becomes soft and starts flowing), and is stretched, thus becoming thinner under the action of the force created by the receiving device. Upon leaving the furnace, the glass is cooled, gradually becoming a solid, and the process of fiber extension practically ceases.

Assuming the workpiece and microcapillary walls to be sufficiently thin, we use quasi-two-dimensional (becoming quasi-one-dimensional in virtue of axial symmetry) equations of dynamics of thin films to describe a steady-state glass flow in the molded fiber (see [1, 5-7]). In the given case they are reduced to the form

$$RhV = Q; \tag{1}$$

$$\rho Q\frac{dV}{dx} = \frac{d}{dx}(\Sigma_{\tau\tau}Rh) - \Sigma_{\theta\theta}h\frac{dR}{dx} + \rho ghR; \tag{2}$$

$$\rho QV\lambda k = \Sigma_{\tau\tau}Rh\lambda k - \Sigma_{\theta\theta}h + 2a(R\lambda k - 1) - \rho gRh\frac{dR}{dx}; \tag{3}$$

$$\rho cQ\frac{dT}{dx} = -(q_{v_1} + q_{v_2})\lambda R; \tag{5}$$

$$\Sigma_{\tau\tau} = 2\mu\left(\frac{2}{\lambda}\frac{dV}{dx} + \frac{V}{\lambda R}\frac{dR}{dx}\right); \tag{6}$$

$$\Sigma_{\theta\theta} = 2\mu\left(\frac{1}{\lambda}\frac{dV}{dx} + \frac{2V}{\lambda R}\frac{dR}{dx}\right); \tag{7}$$

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$$\lambda = \left[1 + \left(\frac{dR}{dx}\right)^2\right]^{1/2};$$
$$k = \lambda^{-3}\frac{d^2R}{dx^2}; \tag{8}$$

$$\mu = \mu_0 \exp\left(\frac{U}{GT}\right). \tag{9}$$

The first equation in (1)-(9) is the continuity equation, the second and the third are the projections of the equation of momentum onto the tangent and normal to the generatrix (liquid glass — viscous Newtonian fluid), and the fourth is the heat transfer equation (heat transfer by conduction along a fiber is negligible). As a rule, $q_{\nu_2} = 0$ may be taken. The pressure in the fiber cavity is assumed to be equal to the external pressure.

In equations of motion (2) and (3), the terms of inertia forces, weight, and surface tension may be neglected. Besides, for thin, gradually becoming thinner fibers $\lambda \approx 1$, $kR \ll 1$; as a result, the third equation gives $\Sigma_{\theta\theta} = 0$. With regard for this, an approximate analytical solution of the system (1)-(9) is constructed by the Laplace method, as it has been done earlier for solid fibers [8], on the assumption that the activation energy in the Arrhenius law for viscosity is high. In the initial fiber section, its radius, wall thickness, velocity, and temperature R_0, h_0, V_0, T_0 are prescribed, while in the terminal section (on the receiving device) a reception rate V_1 is given. As a result, the following approximate analytical solution is obtained in the parametric form:

for the heating zone ($0 \le x \le l$, $T_0 \le T \le T_p$)

$$\bar{R} = \bar{h} = 1 - (1 - m)\exp[\theta(\bar{T} - 1)]; \tag{10}$$

$$x = -\frac{T_p\rho c Q}{q_1 R_0}\{\bar{T} - \bar{T}_0 - \theta^{-1}\ln[1 - (1 - m)\exp[\theta(\bar{T} - 1)]]\}; \tag{11}$$

for the cooling zone ($l \le x \le L$, $T_p \ge T \ge T_1$)

$$\bar{R} = \bar{h} = m - (E^{-1/2} - m)\{\exp[\theta(\bar{T} - 1)] - 1\};$$
$$x = l - \frac{T_p\rho c Q}{q_1 R_0}\left(\frac{q_2}{q_1}\right)^{-1} E^{1/2} \times \tag{12}$$

$$\times \{\bar{T} - 1 - \theta^{-1}\ln[m^{-1}E^{-1/2} - (m^{-1}E^{-1/2} - 1)\exp[\theta(\bar{T} - 1)]]\}. \tag{13}$$

In both regions

$$\bar{V} = \bar{R}^{-1/2}; \tag{14}$$

$$m = \frac{1 - E^{-1/2}\left(\frac{q_2}{q_1}\right)}{1 - \frac{q_2}{q_1}}, \tag{15}$$

where $\bar{R} = R(x)/R_0$; $= h(x)/h_0$; $\bar{V} = V(x)/V_0$; $\bar{T} = T(x)/T_p$; $\bar{T}_0 = T_0/T_p$; $\bar{T}_1 = T_1/T_p$; $\theta = u/(GT_p)$; $E = V_1/V_0$ is the pulling ratio; $q_1 = q_{\nu_1} < 0$ (heating) at $0 \le x \le l$; $q_2 = q_{\nu_1} > 0$ (cooling) at $l \le x \le L$.

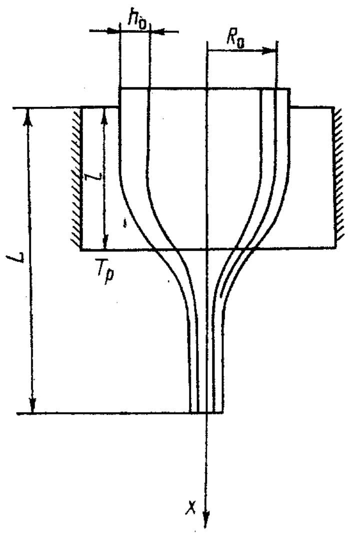

Fig. 1. Schematic drawing of flow.

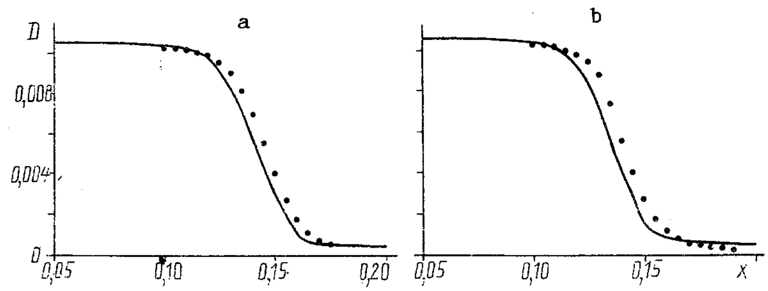

Fig. 2. Configuration of the pulling zone (curves, analytical solution; points, experiment): a) T_0 = 638 K, T_p = 1003 K, T_1 = 673 K, l = 0.16 m; b) T_0 = 673 K, T_p = 1103 K, T_1 = 870 K, l = 0.15 m. D, x, m.

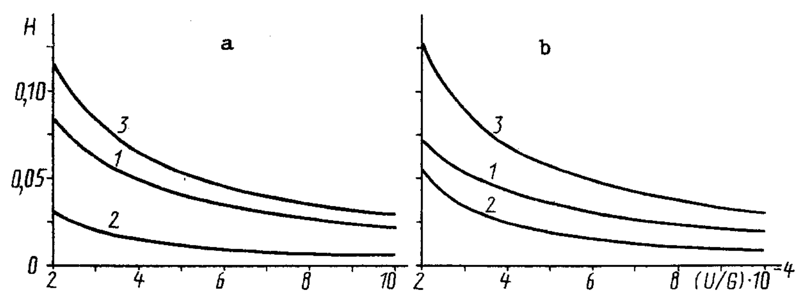

Fig. 3. Transient zone length vs activation energy of viscous flow (a and b correspond to the conditions in Fig. 2; E = 478): 1) H_1, 2) H_2, 3) H = H_1 + H_2. H_1, H_2, H, m; U/G, deg.

The temperatures at the end of the heating zone and on the receiving device to are related the given values as

$$T_p = \frac{U}{2G\ln(m)} + \left[\frac{U^2}{4G^2\ln^2(m)} - \frac{U}{G\ln(m)}\left(T_0 - l\,\frac{q_1R_0}{\rho cQ}\right)\right]^{1/2}; \tag{16}$$

$$l = -\frac{T_p\rho cQ}{q_1R_0}\left[1 - \frac{T_0}{T_p} - \theta^{-1}\ln(m)\right]; \tag{17}$$

$$T_1 = T_p\left[1 - \theta^{-1}\ln(m^{-1}E^{-1/2}) - \frac{(L-l)q_1R_0}{T_p\rho cQE^{1/2}}\frac{q_2}{q_1}\right]. \quad (18)$$

A comparison of (10)-(18) with the results of numerical integration of the system (1)-(8) made in [1] demonstrates a satisfactory accuracy of the approximate analytical solution.

In some cases it is much easier in experiments to determine T_p (with the heating zone length l and temperature T_0 being, naturally, known) as well as T_1 (with the length L known) than heat fluxes q_1 and q_2. In this case the following relations are obtained from (16)-(18):

$$q_1 = -\frac{T_p\rho cQ}{lR_0}\left\{1 - \frac{T_0}{T_p} - \theta^{-1}\ln(m)\right\}; \quad (19)$$

$$\frac{q_2}{q_1} = -\frac{T_p\rho cQE^{1/2}}{q_1R_0(L-l)}\left\{\frac{T_1}{T_p} - 1 + \theta^{-1}\ln(m^{-1}E^{-1/2})\right\}, \quad (20)$$

which allow calculation of q_1 and q_2 by an iteration process. Indeed, prescribing the initial approximation for q_2/q_1, we calculate m with the aid of (15), then find q_1 and a new value of q_2/q_1 from (19) and (20), and so on until the desired accuracy is achieved.

The results obtained allow approximation of a transient zone in which a pulled workpiece undergoes major deformation. For this, we assume that at $0 \le x \le l - H_1$ the glass tube represents a hollow cylinder with the external radius $R_0 + h_0/2$; at $l - H_1 \le x \le l$ it is a hollow truncated cone with base radii $R_0 + h_0/2$ and $R_p + h_p/2$; at $l \le x \le l + H_2$ — a hollow truncated cone with the base radii $R_p + h_p/2$ and $R_1 + h_1/2$; at $l + H_2 \le x \le L$ — a hollow cylinder with base radius $R_1 + h_1/2$. Solving (10)-(15) in the same approximation, we calculate side surface areas of a molded fiber corresponding to the heating and cooling zones

$$S_1 = 2\pi\int_0^l\left(R + \frac{h}{2}\lambda\right)\left\{1 + \left[\frac{d}{dx}\left(R + \frac{h}{2}\lambda\right)\right]^2\right\}^{1/2}dx = -2\pi\frac{\rho cQ}{q_1R_0}\left(R_0 + \frac{1}{2}h_0\right)(T_p - T_0); \quad (21)$$

$$S_2 = 2\pi\int_l^L\left(R + \frac{h}{2}\lambda\right)\left\{1 + \left[\frac{d}{dx}\left(R + \frac{h}{2}\lambda\right)\right]^2\right\}^{1/2}dx = 2\pi\frac{\rho cQ}{q_1R_0}\left(\frac{q_2}{q_1}\right)^{-1}\left(R_0 + \frac{1}{2}h_0\right)(T_p - T_1). \quad (22)$$

On the other hand, each of the areas S_1 and S_2 must be approximately equal to the total lateral surface of the corresponding pair "cylinder–truncated cone," approximating a transient region of the fiber in the heating or cooling zone. From this condition, using (21) and (22), we determine the heights of truncated cones:

$$H_1 = \frac{4C_*(1+m)\sqrt{4C_*^2 + (R_0 + h_0/2)^2(1-m)^3(3+m)}}{(1-m)(3+m)}, \quad (23)$$

herein

$$C_* = \frac{\rho cQ}{q_1R_0}\frac{T_p}{\theta}\ln(m);$$

$$H_2 = -\frac{4D_*E^{1/2}}{m^2E + 2mE^{1/2} - 3} + \quad (24)$$

$$+\sqrt{\frac{4ED_*^2(m^2E+2mE^{1/2}+1)}{(m^2E+2mE^{1/2}-3)^2}-(R_0+h_0/2)^2(m+E^{-1/2})^2\frac{mE^{1/2}-1}{mE^{1/2}+3}},$$

where

$$D_*=-\frac{T_p}{\theta}\ln(m^{-1}E^{-1/2})\frac{\rho cQ}{q_1R_0}\left(\frac{q_2}{q_1}\right)^{-1}.$$

Physically, the dimensions H_1 and H_2 mean that with an accuracy of 5-6% the pulled workpiece deformation begins to develop in the section $x = l - H_1$ and ceases in the section $x = l + H_2$.

Knowing H_1 and H_2, we may evaluate the deformation zone dimensions of a workpiece to be molded as well as an influence of U/G and T_p on these dimensions.

Figure 2 represents a comparison of the above analytical solution of (10)-(15) and (19), (20) with experimental data obtained on a laboratory set-up designed for pulling of microcapillaries from glass tubes at the Institute of Mechanics and Biomechanics of the Bulgarian Academy of Sciences. Parameters had the following values: $D_0 = 10.5 \times 10^{-3}$ m, $h_0 = 10^{-3}$ m, $V_0 = 2.5 \times 10^{-5}$ m/sec, L = 0.3 m, $D_1 = 0.48 \times 10^{-3}$ m (at a given D_1 one may determine E and, respectively, V_1), $\rho = 3 \times 10^3$ kg/m^3, c = 10^3 J/(kg·deg); values of the remaining quantities are given in Fig. 2. We used U/G = 4.7×10^4 deg (U = 93.4 kcal/mole; $\theta = 46.86$ for Fig. 2a and $\theta = 42.61$ for Fig. 2b). The activation energy of viscous flow U is consistent with the data reported in [9]. From Fig. 2 it is seen that at a reasonable value of U the neglect of inertia, gravitation, and surface forces on constructing the analytical solution (10)-(15) is quite justified.

The proposed approximation of the transient zone allows evaluation of furnace dimensions. Approximate dimensions H_1 and H_2 depend on the assumed maximum heating temperature, kinematic process characteristics, physicochemical properties of the treated material, etc.

Figure 3 gives the results of relations (23) and (24) used to calculate H_1, H_2 and $H = H_1 + H_2$ for the main parameters, described above and adopted in physical experiments.

At high activation energies, H_1(U/G) and H_2(U/G) change more slightly. Since the glass stock viscosity essentially depends on the real chemical composition, the proposed approximation also allows evaluation of the variation of transient zone dimensions in dependence on the chemical composition of the glass stock.

The results obtained may be used in designing axisymmetric furnaces with electrical resistance intended for pulling of microcapillaries and threads as well as for simulation of the existing technologies.

NOTATION

R, radius of the median surface of the fiber wall; h, thickness of the fiber wall; V, velocity of longitudinal motion (along the generatrix) of liquid glass in the fiber; Q, volume flow rate divided by 2π; x, longitudinal coordinate along the axis of fiber symmetry; $\Sigma_{\tau\tau}$ and $\Sigma_{\theta\theta}$, longitudinal (along the generatrix) and azimuthal internal stresses in fiber; ρ, glass density; g, gravitational acceleration along axis 0x; λ, elongation of an infinitesimal element of the generatrix as compared to an element of the axis 0x corresponding to it; k, curvature of the generatrix; a, surface tension coefficient; T, temperature; c, heat capacity of glass; $q_{\nu 1}$ and $q_{\nu 2}$, heat fluxes in the direction of external normals to outer and inner sides of the fiber wall; μ, viscosity; U, activation energy of viscous flow; μ_0, preexponential in the Arrhenius law for viscosity; G, gas constant; R_0, h_0, V_0, and T_0, initial radius of the median surface, wall thickness, velocity, and temperature; V_1, fiber reception rate; l, length of the heating zone; T_p, glass temperature at the end of the heating zone (at $x = l$); L, distance between the section, where heating is started, and the receiving device (fiber length); E, pulling ratio; D_0 and D_1, outer diameters of a workpiece at the furnace inlet and outlet; D(x), outer diameter of the stretched tube; H_1 and H_2, heights of truncated cones; S_1 and S_2, areas of lateral surfaces of a shaped microcapillary in the heating and cooling zones; H, length of the transient zone of the fiber. Indices 0, p, and 1 denote R, h, and T referring to the initial section (x = 0), heating zone boundary ($x = l$), and fiber section on the receiving device (x = L).

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